A COMPUTATIONAL FRAMEWORK FOR PREDICTIVE DIGITAL TWINS OF CIVIL ENGINEERING STRUCTURES

MATTEO TORZONI, MARCO TEZZELE, STEFANO MARIANI, ANDREA MANZONI AND KAREN E. WILLCOX
matteo.torzoni@polimi.it

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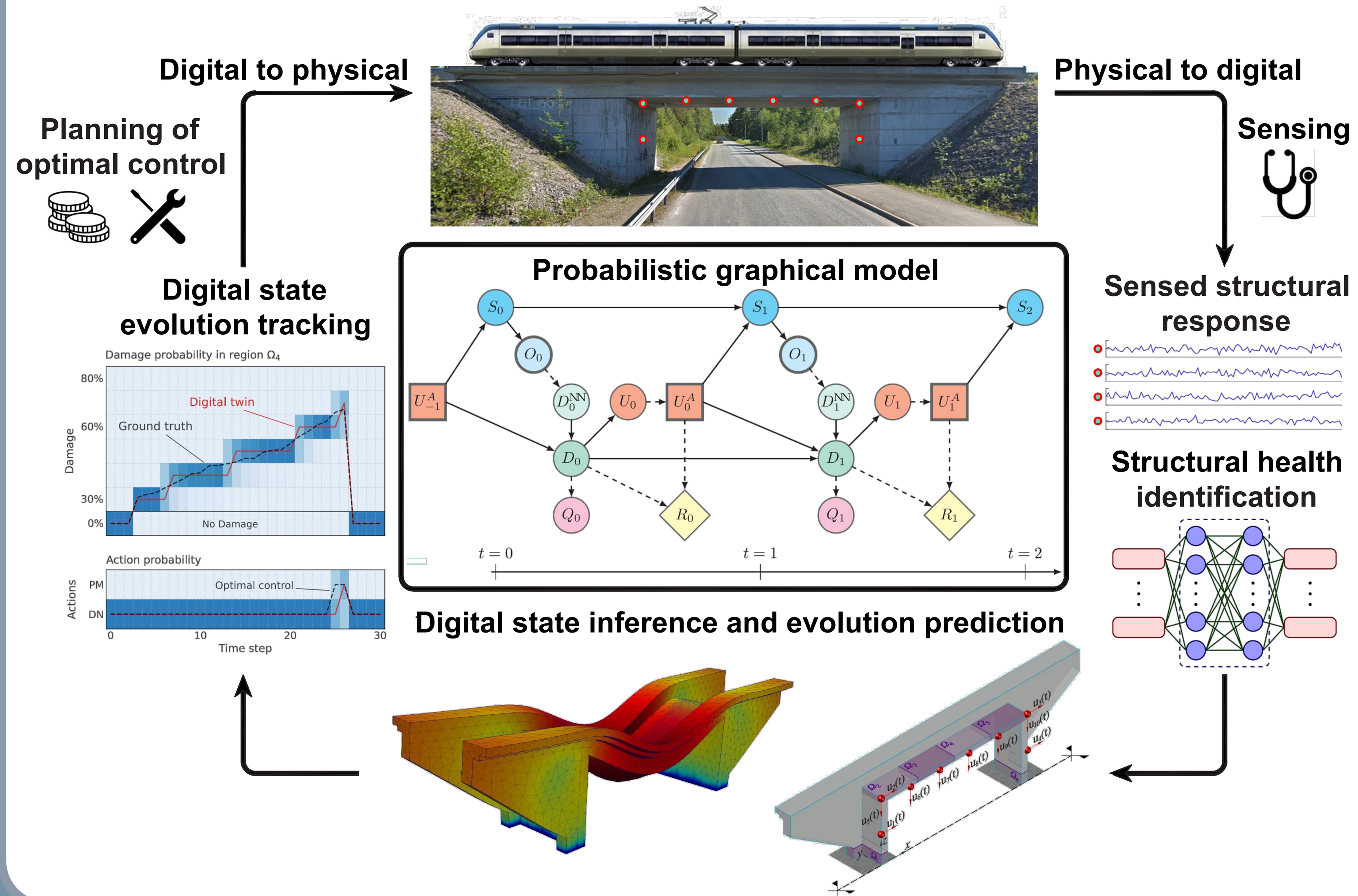
OVERVIEW

Digital twin concept: an appealing opportunity to advance predictive maintenance practices.



Online:

- Asset-twin system encoded using a probabilistic graphical model.
- Sensor data assimilated with DNNs to provide structural health diagnostics.
- Digital twin state continually updated via sequential Bayesian inference.
- Informed optimal planning of actions.

Offline:

- Generate training data via ROMs.
- Learn a control policy (planning).

SIMULATION-BASED DAMAGE IDENTIFICATION

Physics-based numerical model describing the structural dynamic response to applied loadings:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}(\boldsymbol{\mu})\dot{\mathbf{x}}(t) + \mathbf{K}(\boldsymbol{\mu})\mathbf{x}(t) = \mathbf{f}(t,\boldsymbol{\mu}), \ t \in (0,T) \\ \mathbf{x}(0) = \mathbf{x}_0, \\ \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0. \end{cases}$$

- Parameters $\boldsymbol{\mu}$: damage, loads, environment.
- ROM via reduced basis method for parametrized systems (POD): $\mathbf{x}(t,\boldsymbol{\mu}) \approx \mathbf{W}\widehat{\mathbf{x}}(t,\boldsymbol{\mu})$. Galerkin projection:

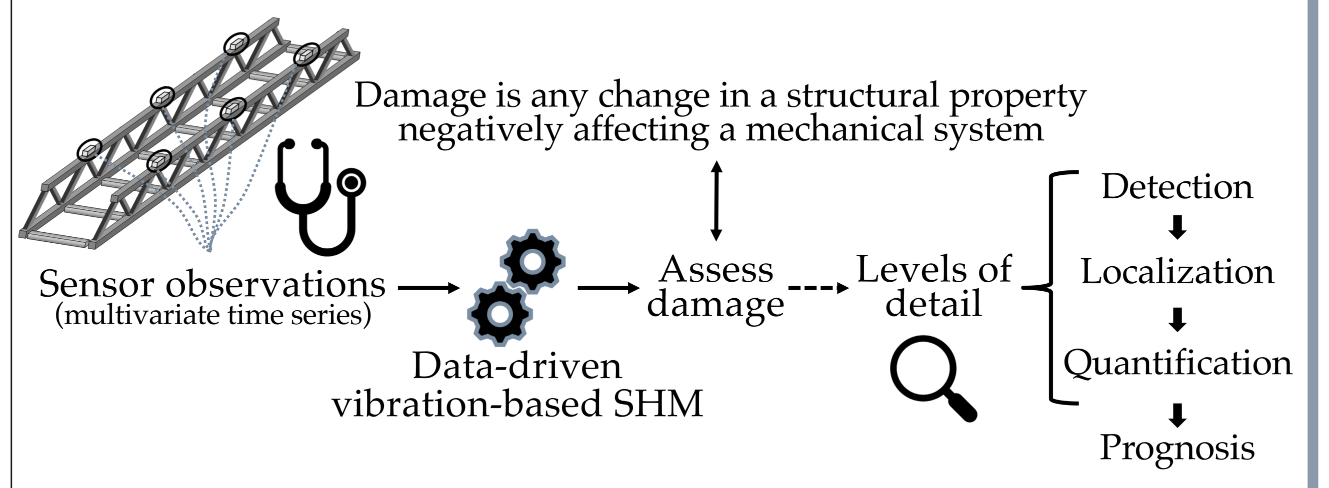
$$\mathbf{M}_r \equiv \mathbf{W}^\top\mathbf{M}\mathbf{W}, \quad \mathbf{C}_r(\boldsymbol{\mu}) \equiv \mathbf{W}^\top\mathbf{C}(\boldsymbol{\mu})\mathbf{W},$$
$$\mathbf{K}_r(\boldsymbol{\mu}) \equiv \mathbf{W}^\top\mathbf{K}(\boldsymbol{\mu})\mathbf{W}, \quad \mathbf{f}_r(t,\boldsymbol{\mu}) \equiv \mathbf{W}^\top\mathbf{f}(t,\boldsymbol{\mu}).$$

- Low-dimensional, low-cost, physics-based model:

$$\begin{cases} \mathbf{M}_r\ddot{\widehat{\mathbf{x}}}(t) + \mathbf{C}_r(\boldsymbol{\mu})\dot{\widehat{\mathbf{x}}}(t) + \mathbf{K}_r(\boldsymbol{\mu})\widehat{\mathbf{x}}(t) = \mathbf{f}_r(t,\boldsymbol{\mu}), \ t \in (0,T) \\ \widehat{\mathbf{x}}(0) = \mathbf{W}^\top\mathbf{x}_0, \\ \dot{\widehat{\mathbf{x}}}(0) = \mathbf{W}^\top\dot{\mathbf{x}}_0. \end{cases}$$

- Compare solution trajectories with sensor recordings.

Structural health monitoring (SHM) workflow.



Simulate sensor data in the presence of damage:

- Damage simulated as a local stiffness reduction of variable magnitude within a set of predefined subdomains.

Train DNNs to solve the SHM problem:

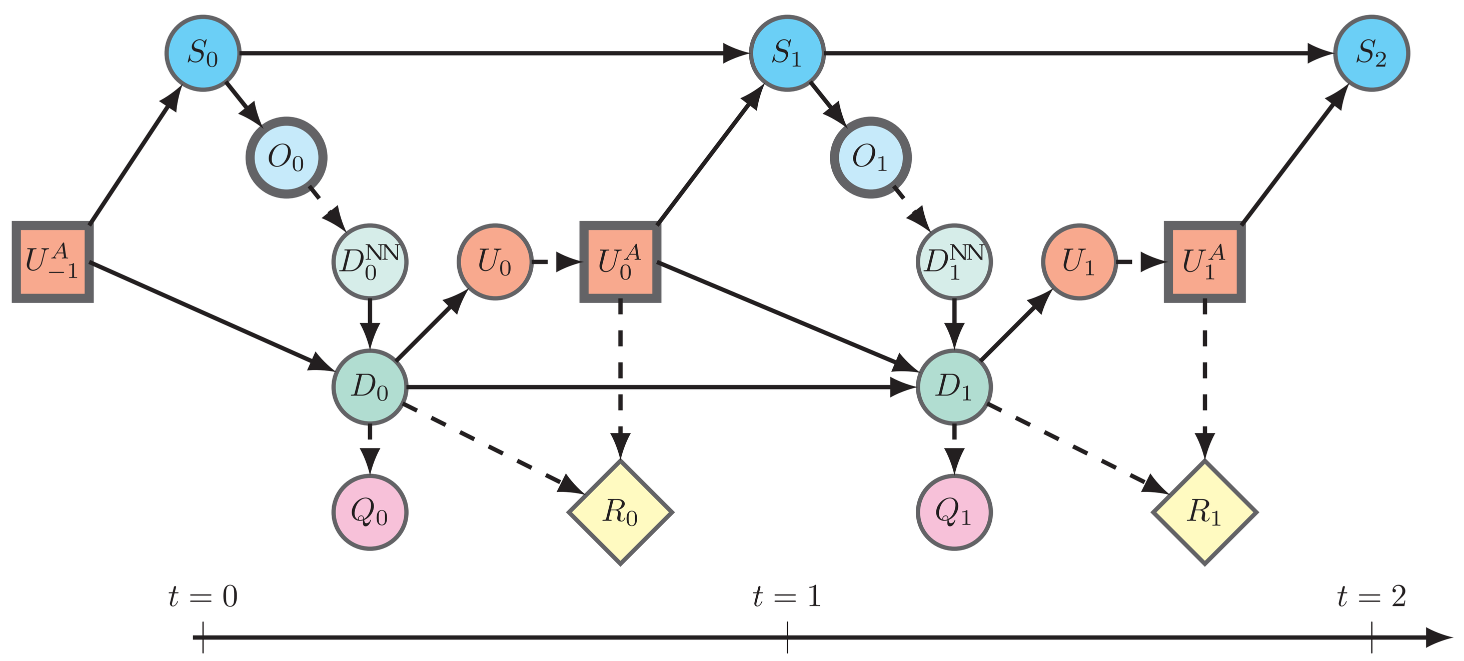
- Damage detection/localization as a classification task.
- Damage quantification as a regression task.

PROBABILISTIC GRAPHICAL MODEL FOR PREDICTIVE DIGITAL TWINS

Involved variables:

Physical space
- Physical state: $S_t \sim p(s_t)$
- Observations: $O_t \sim p(o_t)$
- Control inputs: $U_t \sim p(u_t)$

Digital space
- Digital state: $D_t \sim p(d_t)$
- QoI: $Q_t \sim p(q_t)$
- Reward: $R_t \sim p(r_t)$



Assumptions behind the graph topology:

- Physical state only observable indirectly.
- Markovianity of physical and digital states.

Belief state factorization exploiting the conditional independence structure induced by the graph:

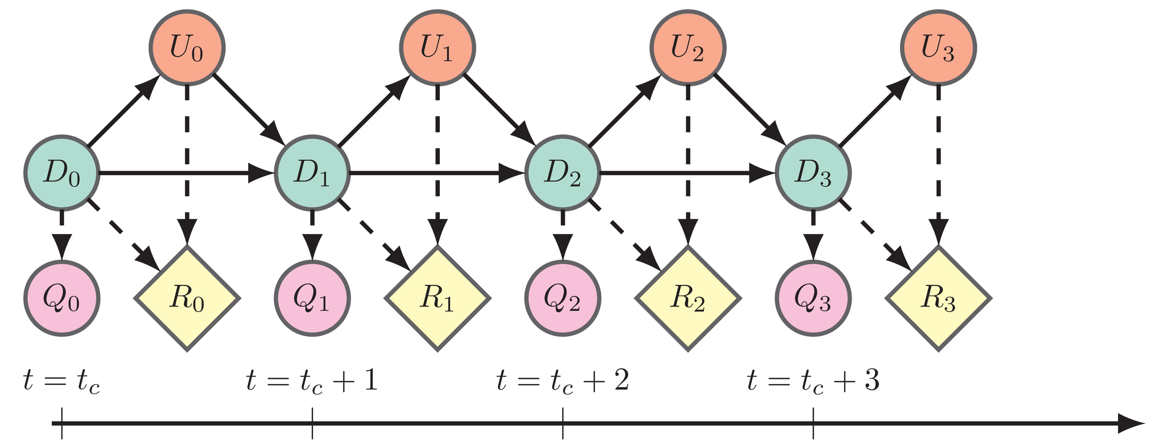
$$p(D_0^{\text{NN}},\dots,D_{t_c}^{\text{NN}},D_0,\dots,D_{t_c},Q_0,\dots,Q_{t_c},R_0,\dots,R_{t_c},U_0,\dots,U_{t_c}|o_0,\dots,o_{t_c},u_0^A,\dots,u_{t_c}^A)$$
$$\propto \prod_{t=0}^{t_c}\left[\phi_t^{\text{data}}\phi_t^{\text{history}}\phi_t^{\text{NN}}\phi_t^{\text{QoI}}\phi_t^{\text{control}}\phi_t^{\text{reward}}\right].$$

Each factor encodes one of the operations carried out within the graph:

$$\phi_t^{\text{data}} = p(O_t = o_t|D_t^{\text{NN}}), \quad \phi_t^{\text{history}} = p(D_t|D_{t-1},U_{t-1}^A = u_{t-1}^A), \quad \phi_t^{\text{NN}} = p(D_t|D_t^{\text{NN}}),$$
$$\phi_t^{\text{QoI}} = p(Q_t|D_t), \quad \phi_t^{\text{reward}} = p(R_t|D_t,U_t^A = u_t^A), \quad \phi_t^{\text{control}} = p(U_t|D_t).$$

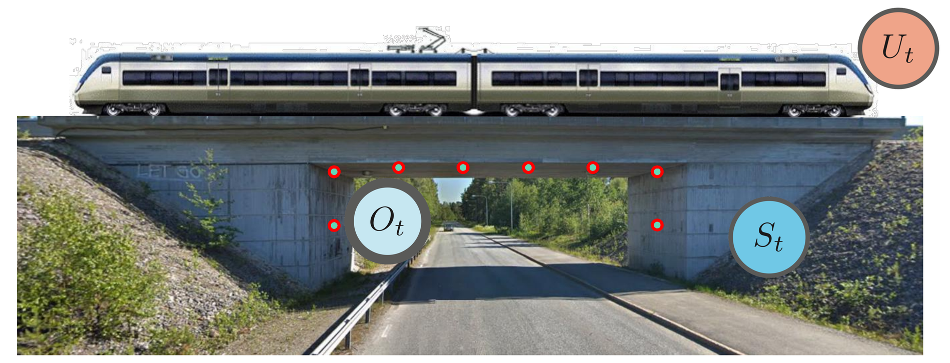
Planning of optimal control: from the updated digital state at the current time t_c, unroll the portion of the graph relative to D_t, Q_t, U_t, and R_t until a prediction time.

- Optimization problem: $\pi(D_t) = \arg\max_\pi \sum_{t=0}^{+\infty}\gamma^t\mathbb{E}[R_t]$.
- Reward function: $R_t(U_t,D_t) = R_t^{\text{control}}(U_t) + \alpha R_t^{\text{health}}(D_t)$.



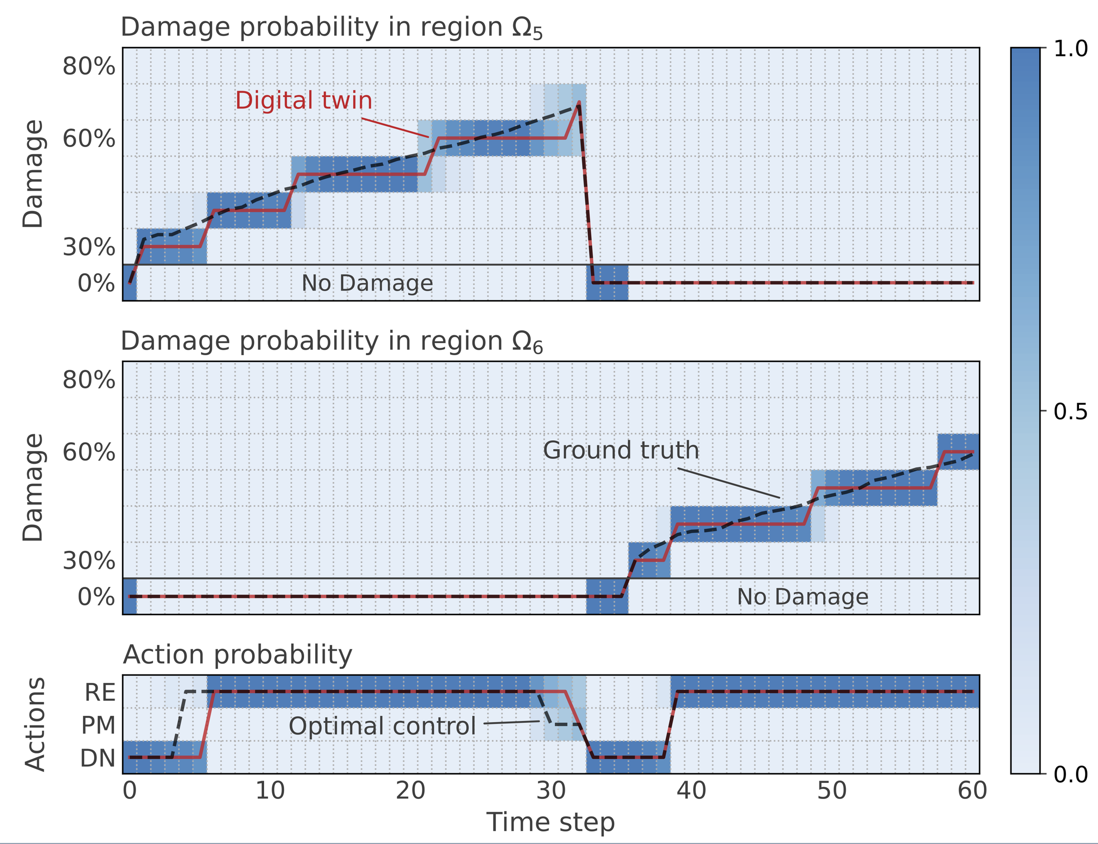
Extend factorization over prediction horizon.

DIGITAL TWIN OF THE HÖRNEFORS RAILWAY BRIDGE

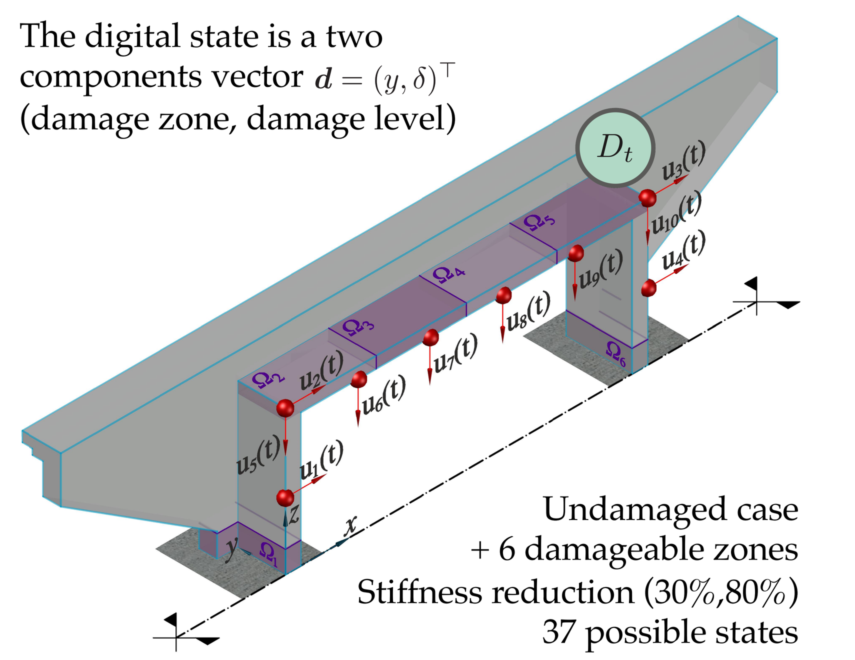


Available control inputs: do nothing (DN), perfect maintenance (PM), restrict operations (RE).

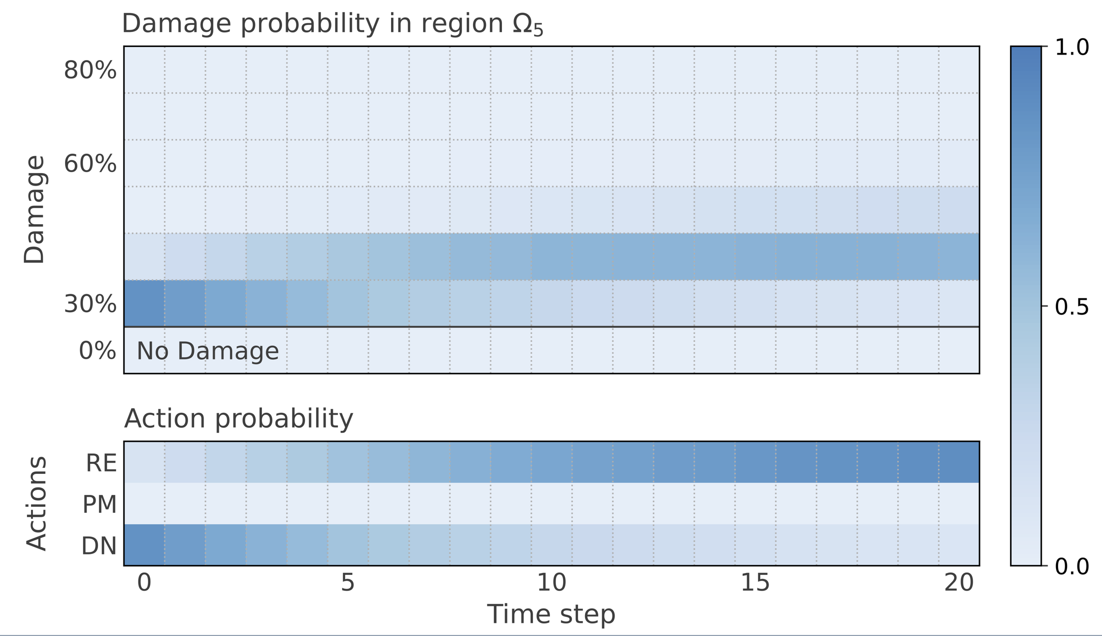
Digital twin simulation: we prescribe a degradation process, and the digital twin is dynamically updated and used to drive management and maintenance planning.



The digital state is a two components vector $\boldsymbol{d} = (y,\delta)^\top$ (damage zone, damage level)



Instance of predicted evolution of digital state and control inputs from the updated digital state at $t_c = 5$.



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